

Cube Difference Labeling of Graph Structures for Medical Imaging and Physiological Network Analysis

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ABSTRACT

A new labeling and a new graph called cube difference labeling and the cube difference is defined. Let G be a (p,q) graph. G is said to have a cube difference labeling if there exists injection $f:V(G)\rightarrow\{0,1,2,\dots,p-1\}$ such that the edge set of G has assigned a weight defined by the absolute cube difference of its end vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph. The cube difference labeling for some special graph families like **Pan graph, Lollipop graph, Barbell graph, Sunlet graph, Sparkler graph, Fan graph, Triangular Snake Graph, Z- P_n graph** are discussed in this paper.

Keywords: *Cube difference labeling, Cube difference graph.*

I. INTRODUCTION

All graph in this paper are simple finite undirected and nontrivial graph $G = (V,E)$ with vertex set V and the edge set E . A function f is a cube difference labeling of a graph G of size n if f is an injection from $V(G)$ to the set $\{0,1,2,\dots,p-1\}$ such that, when each edge uv of G has assigned the weight $|[f(u)]^3-[f(v)]^3|$, the resulting weights are distinct. The notion of square difference labeling was introduced by J.Shima [4]-[6]. Graph labeling can also be applied in areas such as communication network, mobile telecommunications, and medical field. A dynamic survey on graph labeling is regularly updated by Gallian [2] and it is published by Electronic Journal of Combinatory. The notation and terminology used in this paper are taken from [1].

Definition 1.1: Let $G = (V(G),E(G))$ be a graph. G is said to be cube difference labeling if there exist a injection $f:V(G)\rightarrow\{0,1,2,\dots,p-1\}$ such that the induced function $f^*:E(G)\rightarrow\mathbb{N}$ given by $f^*(uv) = |[f(u)]^3-[f(v)]^3|$ is injection.

Definition 1.2: A graph which satisfies the cube difference labeling is called the cube difference graph.

Definition 1.3: The Pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge. It is denoted by P_n .

Definition 1.4: The Lollipop graph is the graph obtained by a Complete graph K_m to a path P_n with a bridge. It is denoted by $L_{m,n}$.

Definition 1.5: The Barbell graph is obtained by connecting two copies of K_n by a bridge. It is denoted by B_n .

Definition 1.6: The Sunlet graph S_n is a graph obtained from a cycle C_n attached a pendent edge at each vertex of the n -cycle. It has $2n$ vertices and $2n$ edges.

Definition 1.7: The Sparkler graph P_m^{+n} is a graph obtained from a path P_m and appending n edges to an end point. It has $m+n$ vertices and $m+n-1$ edges.

Definition 1.8: A fan graph obtained by joining all the vertices of a path P_n to a further vertex, called the Centre. It is denoted by F_n . It has $n+1$ vertices and $2n-1$ edges.

Definition 1.9: The Triangular Snake T_n is obtained the path P_n by replace each of the path by a triangle. It has $2n+1$ vertices and $3n$ edges.

Definition 1.10: In a pair path P_n , i^{th} vertex of a path P_1 is joined with $i+1^{\text{th}}$ vertex of a path P_2 . It is denoted by $Z-P_n$.

II. MAIN RESULT

Theorem: 2.1

The Pan graph P_n admits a Cube difference labeling.

Proof:

Let P_n be a Pan graph. Let $|V(G)| = n+1$ and $|E(G)| = n+1$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,n-1\}$ is defined by

$f(u) = 0$ and $f(u_i) = i+2, 0 \leq i \leq n-1$ and the induced function, $f^*:E(G) \rightarrow N$ is defined by

and here the edge sets are $E_1 = \{u_i u_{i+1} / 0 \leq i \leq n-1\}$ and $E_2 = \{u u_i / i=1\}$

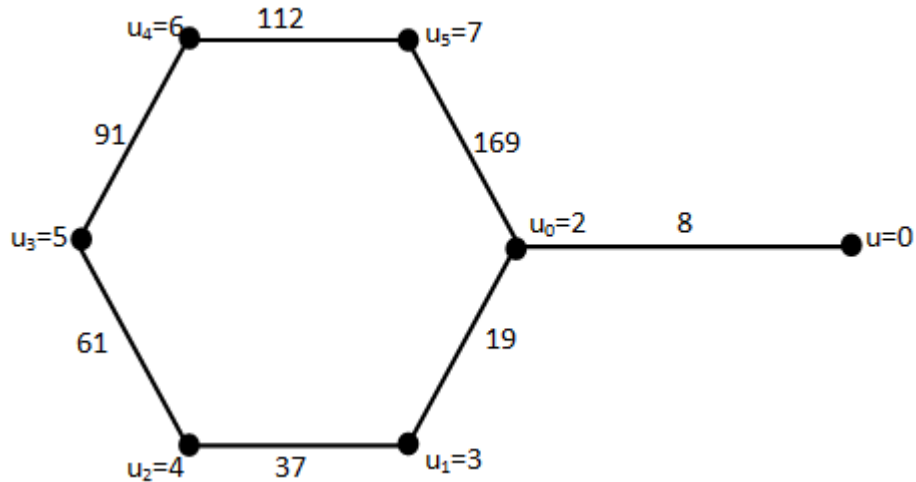
and the edge labeling are,

$$\begin{aligned} \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} |(i+1)^3 - (i+3)^3| \\ &= \bigcup_{i=0}^{n-1} (3i^2 + 15i + 19) \\ &= \{19, 37, 61, \dots\} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f^*(u u_0) &= (i+2)^3, i=0 \\ &= 8. \end{aligned}$$

Here all the edges are distinct. Hence, the Pan graph P_n admits a Cube difference labeling.

Example 2.2: The Pan graph P_6 is a cube difference graph.



Theorem: 2.3

The Lollipop graph $L_{m,n}$ admits a Cube difference labeling.

Proof:

Let $L_{m,n}$ be a Lollipop graph. Let $|V(G)| = m+n$ and $|E(G)| = m+n+2$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,n-1\}$ is defined by

$f(u_i) = i, 0 \leq i \leq n-1$ and $f(v_i) = i+1, n-1 \leq i \leq 2(m-1)$ the induced function,

$f^*:E(G) \rightarrow N$ is defined by and here the edge sets are $E_1 = \{u_i u_{i+1} / 0 \leq i \leq n-1\}$ and $E_2 = \{v_i v_{i+1} / n \leq i \leq 2(m-1)\}$,

$E_3 = \{v_i v_{i+2} / i=3\}$ and $E_4 = \{v_{i+2} v_{i+4} / i=2\}$ and the edge labeling are,

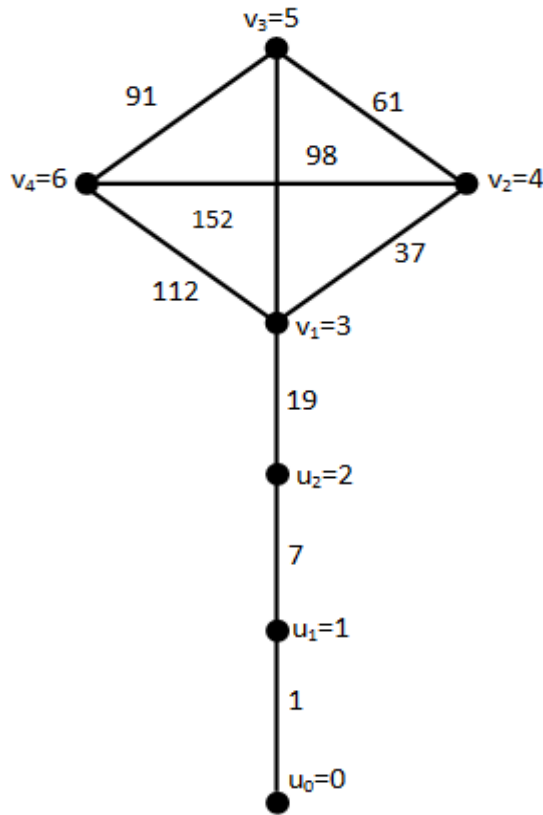
$$\begin{aligned} \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} |i^3 - (i+1)^3| \\ &= \bigcup_{i=0}^{n-1} (3i^2 + 3i + 1) = \\ &= \{1, 7\}. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f^*(v_i v_{i+1}) &= \bigcup_{i=1}^m |(f(v_i))^3 - (f(v_{i+1}))^3| \\ &= \bigcup_{i=1}^m (3i^2 + 3i + 7). \end{aligned}$$

$$\begin{aligned}
 &= \{19, 37, 61, 91, 112\}. \\
 \text{(iii)} \quad f^*(v_i v_{i+2}) &= |(f(v_i))^3 - (f(v_{i+2}))^3| \\
 &= |(i)^3 - (i+2)^3| \\
 &= 6i^2 + 24i + 26, \quad i=2 \\
 &= 98 \\
 \text{(iv)} \quad f^*(v_{i+1} v_{i+3}) &= |(f(v_{i+1}))^3 - (f(v_{i+3}))^3| \\
 &= |(i+2)^3 - (i+4)^3| \\
 &= 6i^2 + 36i + 56, \\
 &= 152.
 \end{aligned}$$

Here all the edges are distinct. Hence, the Lollipop graph $L_{m,n}$ admits a Cube difference labeling.

Example 2.4: $L_{4,3}$



Theorem: 2.5

The Barbell graph B_n admits a Cube difference labeling.

Proof:

Let B_n be the Barbell graph. Let $|V(G)|=2n$ and $|E(G)|=2n+1$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,2n-1\}$ is defined by $f(u_i)=i+1, 0 \leq i \leq 2n-1$. and induced function $f^*:E(G) \rightarrow N$ is defined by, and here the sets are,

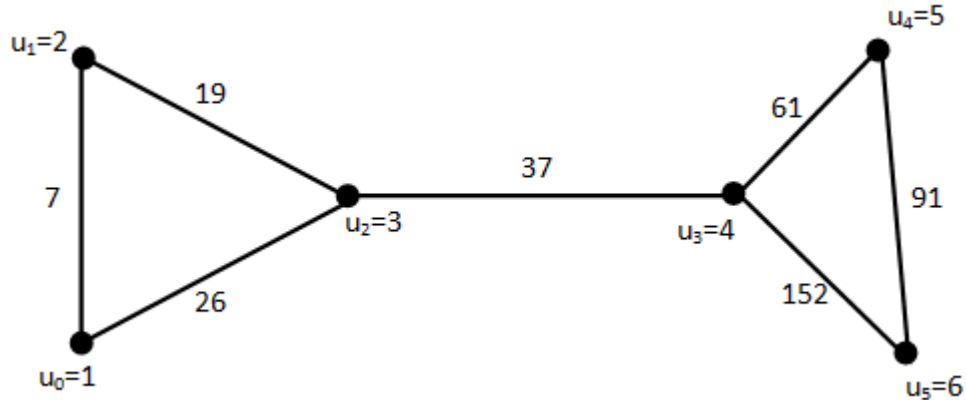
$$E_1 = \{u_i u_{i+1} / 0 \leq i \leq n-1\} \text{ and } E_2 = \{u_i u_{i+2} / i=1\} \text{ and } E_3 = \{u_{i+2} u_{i+4} / i=2\}.$$

$$\begin{aligned}
 \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\
 &= \bigcup_{i=0}^{n-1} |(i+1)^3 - (i+2)^3| \\
 &= \bigcup_{i=0}^{n-1} (3i^2 + 9i + 7) \\
 &= \{1, 7, 19, 37, \dots, 91\} \\
 \text{(ii)} \quad f^*(u_i u_{i+2}) &= |i^3 - (i+2)^3| \\
 &= 6i^2 + 12i + 8, \quad i=1 \\
 &= 26
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad f^*(u_{i+2}u_{i+4}) &= |(f(u_{i+2}))^3 - (f(u_{i+4}))^3| \\
 &= |(i+2)^3 - (i+4)^3| \\
 &= 6i^2 + 36i + 56, \quad i=2 \\
 &= 152.
 \end{aligned}$$

Hence all the edges are distinct. Hence the graph B_n admits a Cube difference labeling.

Example 2.6: The Barbell graph B_3 is a Cube difference graph



Theorem: 2.7

The Sunlet graph S_n admits a Cube difference labeling.

Proof:

Let S_n be a Sunlet graph. Let $|V(G)|=2n$ and $|E(G)|=2n$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,2n-1\}$ is defined by $f(u_i)=i$, $0 \leq i \leq 2n-1$

and the induced function $f^*:E(G) \rightarrow N$ is defined by, and here the sets are,

$E_1 = \{u_i u_{i+1} / 0 \leq i \leq n-1\}$ and $E_2 = \{u_n u_0\}$

$E_3 = \{u_i u_{n+i} / 0 \leq n+i \leq 2n-1\}$ and the edge labeling are,

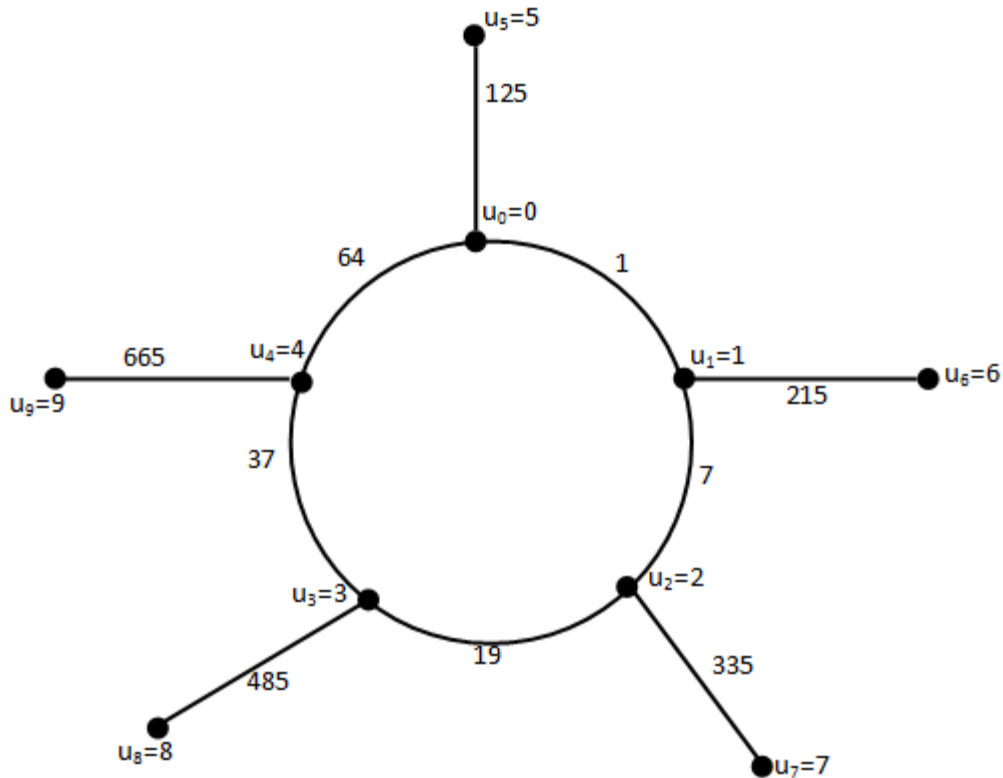
$$\begin{aligned}
 \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\
 &= \bigcup_{i=0}^{n-1} (3i^2 + 3i + 1) \\
 &= \{1, 7, 19, 37\}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad f^*(u_{n-1} u_0) &= (n-1)^3 \\
 &= 64.
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad f^*(u_i u_{n+i}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{n+i}))^3| \\
 &= \bigcup_{i=0}^{n-1} (15i^2 + 75i + 125) \\
 &= \{125, 215, 335, 485, 665\}
 \end{aligned}$$

Here all the edges are distinct. Hence the Sunlet graph S_n admits a Cube difference labeling.

Example 2.8: The Sunlet graph S_5 is a Cube difference graph.



Theorem: 2.9

A Sparkler graph P_m^{+n} admits a Cube difference labeling.

Proof:

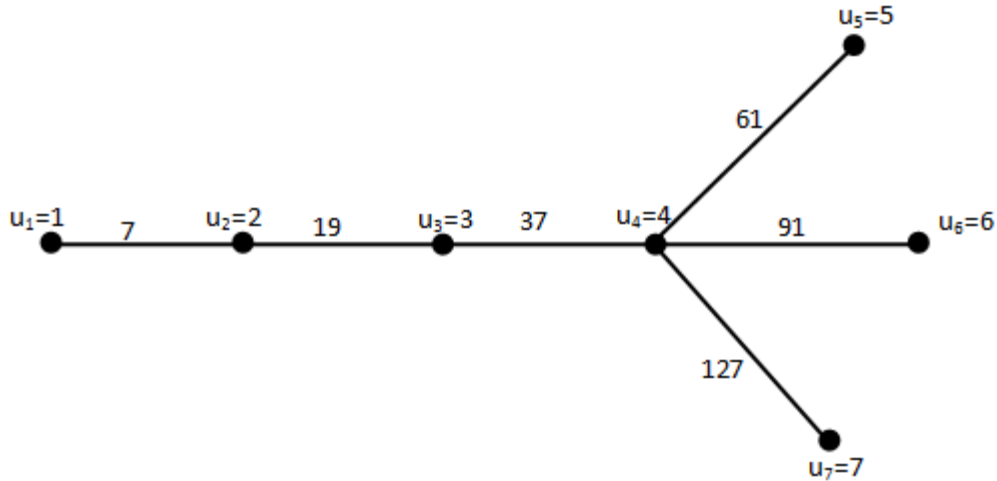
Let P_m^{+n} be a Sparkler graph. Let $|v(G)|=m+n$ and $|E(G)|=m+n-1$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,n-1\}$ is defined by $f(u_i)=i$, $1 \leq i \leq m$ and $f(u_j)=m+1$, $m+1 \leq j \leq 2n+1$, and the induced function, $f^*:E(G) \rightarrow N$ is defined by, and here the sets are, $E_1=\{u_i u_{i+1} / 1 \leq i \leq m-1\}$, $E_2=\{u_i v_j / i=m, m+1 \leq j \leq 2n+1\}$ and the edge labeling are

- (i) $f^*(u_i u_{i+1}) = \bigcup_{i=1}^m |(f(u_i))^3 - (f(u_{i+1}))^3|$
 $= \bigcup_{i=1}^m (3i^2 + 3i + 1)$
 $= \{7, 19, 37\}$
- (ii) $f^*(u_i u_j) = |(f(u_i))^3 - (f(v_j))^3|$, $i=m$ and $m+1 \leq j \leq n$
 $= \bigcup_{i=m+1}^{2n+1} (3i^2 + 3i + 1)$
 $= \{61, 91, 127\}$

Here all the edges are distinct. Hence the Sparkler graph P_m^{+n} admits a Cube difference labeling.

Example 2.10: The Sparkler graph P_4^{+3} is a Cube difference graph.



Theorem: 2.11

The Fan graph F_n admits a Cube difference labeling.

Proof:

Let F_n be a Fan graph. Let $|V(G)|=n+1$ and $|E(G)|=2n-1$.

The mapping $f:V(G) \rightarrow \{0,1,2,\dots,n-1\}$ is defined by $f(u)=0$ and $f(u_i)=i$, $1 \leq i \leq n$

and the induced function $f^*:E(G) \rightarrow \mathbb{N}$ is defined by, and here the sets are,

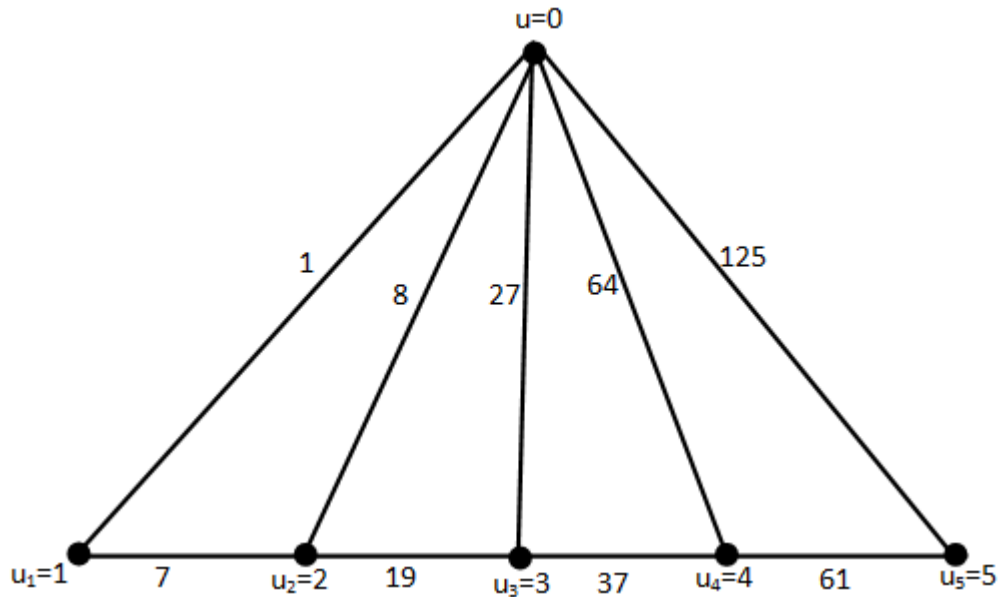
$E_1=\{u_i u_{i+1} / 1 \leq i \leq n-1\}$ and $E_2=\{u u_i / 1 \leq i \leq n\}$ and the edge labelings are,

$$\begin{aligned} \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=1}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\ &= \bigcup_{i=1}^{n-1} (3i^2 + 3i + 1) \\ &= \{7, 19, 37, 61\} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f^*(u u_i) &= \bigcup_{i=1}^n |(f(u))^3 - (f(u_i))^3| \\ &= \bigcup_{i=1}^n (i)^3 \\ &= \{1, 8, 27, 64, 125\} \end{aligned}$$

Here all the edges are distinct. Hence the Fan graph F_n admits a Cube difference labeling.

Example 2.12: The Fan graph F_5 is a Cube difference graph.



Theorem: 2.13

A Triangular Snake graph T_n admits a Cube difference labeling.

Proof:

Let T_n be a Triangular Snake graph. Let $|V(G)|=2n+1$ and $|E(G)|=3n$. The mapping $f:V(G) \rightarrow \{0,1,2,\dots,2n-1\}$ is defined by $f(u_i)=2i$, $0 \leq i \leq n-1$ and $f(v_i)=2i+1$, $0 \leq i \leq n-1$ and the induced function, $f^*:E(G) \rightarrow N$ is defined by, and here the sets are, $E_1=\{v_i v_{i+1} / 0 \leq i \leq n-1\}$, $E_2=\{u_i v_i / 0 \leq i \leq n-1\}$ and $E_3=\{u_i v_{i+1} / 0 \leq i \leq n-1\}$ and the edge labelings are,

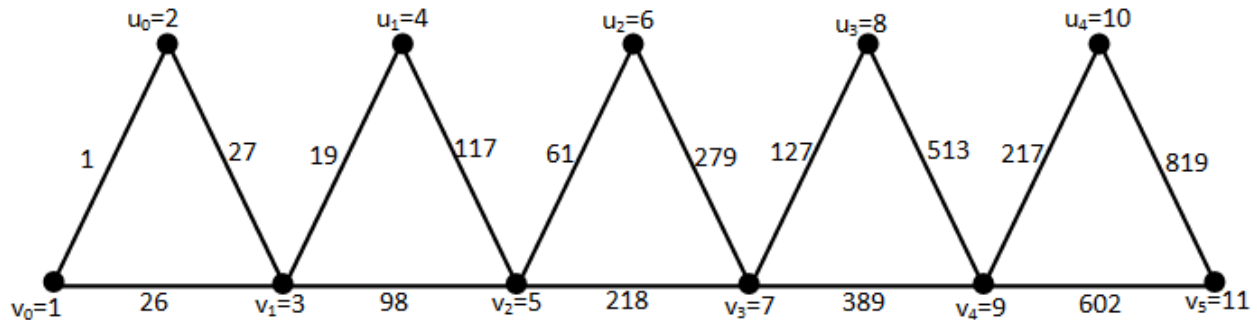
$$\begin{aligned} \text{(i)} \quad f^*(v_i v_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(v_i))^3 - (f(v_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} |(2(i+1))^3 - (2(i+1)+1)^3| \\ &= \bigcup_{i=0}^{n-1} (24i^2 + 48i + 26) \\ &= \{26, 98, 218, 386, 602\}. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f^*(u_i v_i) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(v_i))^3| \\ &= \bigcup_{i=0}^{n-1} |(2i)^3 - (2i+1)^3| \\ &= \bigcup_{i=0}^{n-1} (12i^2 + 6i + 1) \\ &= \{1, 19, 61, 127, 217\} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad f^*(u_i v_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(v_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} (36i^2 + 54i + 27) \\ &= \{27, 117, 279, 513, 819\} \end{aligned}$$

Here all the edges are distinct. Hence the Triangular Snake graph T_n admits a Cube difference labeling.

Example 2.14: The Triangular Snake graph T_5 is a Cube difference graph.



Theorem: 2.15

The $Z-P_n$ graph admits a Cube difference labeling.

Proof:

Let $Z-P_n$ be a graph. Let $|V(G)|=2n$. The mapping $f:V(G) \rightarrow \{0,1,2,\dots,2n-1\}$ is defined by $f(u_i)=2i$, $0 \leq i \leq n-1$ and $f(v_i)=2i+1$, $0 \leq i \leq n-1$ and the induced function $f^*:E(G) \rightarrow N$ is defined by, and here the sets are, $E_1=\{u_i u_{i+1} / 0 \leq i \leq n-1\}$, $E_2=\{v_i v_{i+1} / 0 \leq i \leq n-1\}$ and $E_3=\{v_i u_{i+1} / 0 \leq i \leq n-1\}$ and the edges labelings are

$$\begin{aligned} \text{(i)} \quad f^*(u_i u_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(u_i))^3 - (f(u_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} (24i^2 + 24i + 8) \\ &= \{8, 56, 152, 296\} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad f^*(v_i v_{i+1}) &= \bigcup_{i=0}^{n-1} |(f(v_i))^3 - (f(v_{i+1}))^3| \\ &= \bigcup_{i=0}^{n-1} (24i^2 + 48i + 26) \\ &= \{26, 98, 218, 386\} \end{aligned}$$

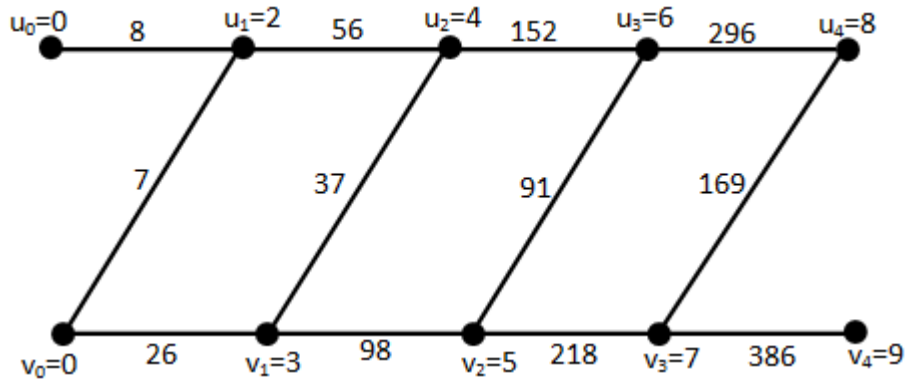
$$\text{(iii)} \quad f^*(v_i u_{i+1}) = \bigcup_{i=0}^{n-1} |(f(v_i))^3 - (f(u_{i+1}))^3|$$

$$= \bigcup_{i=0}^{n-1} (12i^2 + 18i + 7)$$

$$= \{7, 37, 91, 169\}$$

Here all the edges are distinct. Hence $Z-P_n$ admits a Cube difference labeling.

Example 2.16: The $Z-P_5$ graph is a Cube difference graph.



III. CONCLUSION

In this paper the Special graphs, are investigated for the Cube difference labeling. This labeling can be verified for some other graphs.

REFERENCE

1. Frank Harary, *Graph theory*, Narosa Publishing House(2002).
2. J A Gallian, *A dynamic survey of graph labeling. The Electronics journal of Coimbinatories*, 17(2010) # DS6.
3. J.Shima "Square sum labeling for some middle and total graphs" *International Journal of Computer Applications (0975-08887) Volume 37-No.4 January 2012*.
4. J.Shima "Square difference labeling for some path, fan and gear graphs" *International Journal of Scientific and Engineering Research volume 4, issue 3, March -2013, ISSN 2229-5518*.
5. J.Shima "Some Special types of Square difference graphs" *International Journal of Mathematics archives – 3(6), 2012, 2369-2374 ISSN 2229-5046*.
6. J.Shima "Square difference labeling for some graphs" *International Journal of Computer Applications (0975-08887) Volume 44-No.4, April 2012*.
7. J.Shima "Cube difference labeling of Some graphs" *International Journal of Engineering Science and Innovative Research. Volume 2, Issue 6, November 2013, ISSN 2319-5967*.