Time-Dependent Schrödinger Equation for Two-Level Systems in the Analysis of Transverse Relaxation Time for MRI and Biomedical Signal Applications

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ABSTRACT

Transverse relaxation time is important in magnetic resonance imaging (MRI). It is important to construct new models to find new parameters that control imaging quality. In this work Schrodinger Equation in energy space for two level systems was used to find transverse relaxation time. By suggesting sine and cosine beside complex solutions a useful expression for transverse relaxation time was found. When electric interaction dominate, i.e. for dielectric materials the transverse relaxation time depends on the electric dipole moment. However the magnetic materials having magnetic spin and magnetic dipole moment, it depends on the internal field as well as spin quantum number.

***Key Words**: transverse relaxation time, energy wave function, electric dipole moment, internal magnetic field, spins quantum number.*

I. INTRODUCTION

Magnetic resonance imaging (MRI) is one of the most popular techniques which is widely used in medical diagnostic .It gives good image quality since the image contrast depends on three parameters. These parameters are longitudinal, transverse relaxation time beside the so called spin density [1,2].The understanding of these parameters needs understanding the so called nuclear magnetic resonance. In this process the application of external magnetic field splits protons energy levels into two sub levels according to their spin orientation [3]. The protons having magnetic spin opposite to the external field occupies the higher splatter energy level. The protons with magnetic spin moment pointing in the direction of the external magnetic field occupy the lower spited level [4]. The differences between the two levels are proportional to the strength of the external magnetic field. The organ image is formed by applying microwave of specific frequency, then apply sinusoidal variable magnetic field which change the energy different between the two splitter levels. When the energy difference equals the microwave photon energy resonance occurs and the photons are absorbed. The absorption rate normal tissue is different from that abnormal one [5, 6].In conventional (MRI) theories the image contrast depends on longitudinal, transverse relaxation time as well as the density of hydrogen atoms. These also depend on material properties as well as external and internal magnetic field. To improve image quality one need new models and trend this can discover new mechanism that control image contrast, sensitivity and detection limit. Different attempts were made so as to explain MRI on new basis [7, 8, and 9], but one needs more new trends are required. This is the aims of this work is to use the Schrodinger Equation in the momentum space to find useful expression for relaxation time.

Generalized Energy Wave Function for Tow Level System

Consider the time dependent Schrödinger equation for tow level system

$$i\hbar \dot{C}_k = \sum_{n=1}^{2} e^{i\omega_{Kn}t_{kn}} H^{\backslash}_{kn} C_n \qquad (1)$$

Where $C_k(t)$ stands for the wave function in the energy space.

The equations for k=1 and k=2 are given respectively to be

$$i\hbar \dot{C}_1 = H'_{11}C_1 + e^{i\omega_2\tau} H^1_{12} C_2 \qquad (2)$$

$$i\hbar \dot{C}_2 = e^{i\omega_{21}t} H_{21}^1 C_1 + H'_{22} C_2 \quad (3)$$

Where

$$\omega_{12} = \omega_1 - \omega_2 \;\; \omega_{21} = \omega_2 - \omega_1 \quad (4)$$

Differentiating (2) yields:

$$i\hbar \ddot{C}_1 = H'_{11} \dot{C}_1 + i\omega_{12} e^{i\omega_{12}t} H'_{12} C_2 + e^{i\omega_{12}t} H'_{12} \dot{C}_2 \quad (5)$$

Multiplying (5) by $i\hbar$ yields

$$-\hbar^2 \ddot{C}_1 = i\hbar H'_{11} \dot{C}_1 - \hbar\omega_{12} e^{i\omega_{1212}t} H'_{12} C_2 + i\hbar e^{i\omega_{12}t^2} H'_{12} \dot{C}_2 (6)$$

Substituting (3) in (6)

$$-\hbar^2 \ddot{C}_1 = i\hbar H'_{11} \dot{C}_1 - \hbar\omega_{12} e^{i\omega_{12}\tau} H'_{12} C_2 + e^{i\omega_{12}t} H'_{12} [e^{i\omega_{21}t} H'_{21} C_1 + H'_{22} C_2]$$

$$-\hbar^2 \ddot{C}_1 = i\hbar H'_{11} \dot{C}_1 - \hbar\omega_{12} e^{i\omega_{12}\tau} H'_{12} C_2 + H'_{12} H'_{21} C_1 + e^{i\omega_{12}t} H'_{12} H'_{22} C_2 (7)$$

To eliminate C_2 from (7) one can utilize eq. (2) to get

$$e^{i\omega_{12}\tau} H'_{12} C_2 = i\hbar \dot{C}_1 - H'_{11} C_1$$

Using this equation in equation (7) yields:

$$-\hbar^2 \ddot{C}_1 = i\hbar H_{11}^1 \dot{C}_{11} - i\hbar^2 \omega_{12} \dot{C}_1 + \hbar\omega_{12} H'_{11} C_1 + H'_{12} H'_{21} C_1 + i\hbar H'_{22} \dot{C}_1 - H'_{11} H'_{22} C_1$$

$$-\hbar^2 \ddot{C}_1 = i\hbar\left[H'_{11} - \hbar\omega_{12} + H'_{22}\right]\dot{C}_1 + \left[\hbar\omega_{12} H'_{11} + H'_{12} H'_{21} - H'_{11} H'_{22}\right]C_1 (8)$$

$$-\hbar^2 \ddot{C}_1 = i\hbar\left[H'_{11} + H'_{22} - \hbar\omega_{12}\right]\dot{C}_1 + \left[H'_{12} H'_{21} - H'_{11} H'_{22} + \hbar\omega_{12} H'_{11}\right]C_1 (9)$$

Now let

$$a = -\hbar^2, b = i\hbar\left[H'_{11} + H'_{22} - \hbar\omega_{12}\right]$$

$$C = H'_{12} H'_{22} - H'_{11} H'_{22} + \hbar\omega_{12} H'_{11} \quad (10)$$

The equation (9) can be reduced to the form:

$$a\ddot{C}_1 = b\dot{C}_1 + CC_1 \quad (11)$$

The parameters a, b and c are constant in time. Thus one can try the solution of the form:

$$C_1 = Ae^{-\beta t} \sin \omega\tau \quad (12)$$

$$\dot{C}_1 = -\beta A e^{-\beta\tau} \sin\omega\tau + \omega A e^{-\beta\tau} \cos\omega\tau$$

$$\ddot{C}_1 = \beta^2 A e^{-\tau} \sin\omega\tau - 2\beta A e^{-\beta\tau} \cos\omega\tau - \omega^2 A e^{-\beta\tau} \cos\omega\tau - \omega^2 A e^{-\beta} \sin\omega\tau =$$
$$(\beta^2 - \omega^2) A e^{-\beta\tau} \sin\omega\tau - 2\beta\omega A e^{-\beta\tau} \cos\omega\tau \quad (13)$$

Sub. (12) and (13) in (11) yields:

$$a(\beta^2 - \omega^2)\sin\omega\tau - 2a\beta\omega\cos\omega\tau = -b\beta\sin\omega\tau + b\omega\cos\omega t + C\sin\omega t$$

Equating the coefficients at sin $\omega\tau$ and cos ωt on both sides, one gets:

$$a(\beta^2 - \omega^2) = -b\beta + C \quad (14)$$

Thus:

$$2a\beta\omega = b\omega \quad (15)$$

Hence (15) and (10) yields:

$$\beta = \frac{-b}{2a} = \frac{b}{2\hbar^2} \quad (16)$$

To find the frequency ω equation (16) is inserted in equation (14) to get

$$\frac{ab^2}{4a^2} - a\omega^2 = \frac{+b^2}{2a} + C$$

$$-a\omega^2 = \frac{+b^2}{4a} + C \quad (17)$$

Utilizing the expression for a in equation (10) yields:

$$\omega^2 = \frac{-b^2}{4a} + \frac{C}{-a} \quad (18)$$

$$\omega^2 = \frac{C}{\hbar^2} - \frac{b^2}{4\hbar^4}$$

$$\omega = \sqrt{\frac{C}{\hbar^2} - \frac{b^2}{4\hbar^4}} = \frac{1}{2\hbar^2}\sqrt{4\hbar^2 C - b^2}$$

The probability that state E_1 is occupied is given by

$$|C_1|^2 = C_1 C_1^* = A^2 e^{-2\beta t} \sin^2 \omega t \quad (19)$$

The energy level is empty at $t = o$ since sin0=0

Its occupation is a maximum when:

$$\sin \omega\tau = 1$$

$$\omega\tau = \frac{\pi}{2}$$

$$\tau = \frac{\pi}{2\omega} = \frac{\pi}{4\pi f} = \frac{1}{4f} \quad (20)$$

Equation (11) can be solved by suggesting the solution

$$C_1 = Ae^{(-\beta + i\omega)t} \quad (21)$$

Therefore:

$$\dot{C}_1 = +(-\beta + i\omega)C_1, \ddot{C}_1 = +(-\beta + i\omega)^2 C_1 \quad (22)$$

Inserting (21) and (22) in (11) yields:

$$a(-\beta + i\omega)^2 = b(-\beta + i\omega) + C$$

$$a(\beta^2 - \omega^2 - 2\beta\omega i) = -b\beta + C + \omega bi \quad (23)$$

Equating real and imaginary parts yields:

$$a(\beta^2 - \omega^2) = -b\beta + C \quad (24)$$

$$-2a\beta\omega = \omega b \quad (25)$$

Hence an equation (10) and (14) reads:

$$\beta = -\frac{b}{2a} = \frac{b}{2\hbar^2} \quad (26)$$

$$\omega^2 = \beta^2 + \frac{b}{a}\beta - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{b^2}{2a^2} - \frac{c}{a} = \frac{-b^2}{4a^2} - \frac{c}{a}$$

$$\omega^2 = \frac{c}{\hbar^2} - \frac{b^2}{4\hbar^4} = \frac{1}{4\hbar^4}\left[4\hbar^2 c - b^2\right]$$

Thus the frequency is given by:

$$\omega = \frac{1}{2\hbar}\sqrt{4\hbar^2 c - b^2} \qquad (27)$$

The probability of occupation of the energy level E_1 can thus be given according to equation (21) to be

$$\left|C\right|^2 = C_1 - C_1^* = A^2 e^{-2\beta t} \qquad (28)$$

The normalization condition for C_1 is given to be

$$\int_{\circ}^{\infty} \left|C\right|^2 dt = 1$$

$$\frac{A^2}{-2\beta}\left[e^{-2\beta\tau}\right]_{\circ}^{\infty} = 1$$

$$-\frac{A^2}{2\beta}\left[e^{-\infty} - e^{-o}\right] = 1$$

$$\frac{A^2}{2\beta} = 1 \qquad A = \sqrt{2\beta} \qquad (29)$$

There for to equation (21)

$$\therefore C_1 = \sqrt{2\beta}e^{-\beta t}\ \mathrm{e}^{i\omega t} \qquad (30)$$

The occupation probability for
E_1 is thus

$$\left|C_1\right|^2 = 2\beta e^{-2\beta t} \qquad (31)$$

It is clear that at t = 0
The probability is maximum

$$\left|C\right|^2 = 2\beta = \max imum \qquad (32)$$

But the probability is near to zero or very small when:

$$\tau = \frac{1}{2\beta} \qquad (33)$$

The term β can be found with the aid of equations (26) and (10) to be

$$\beta = \frac{b}{2\hbar^2} = \frac{i\hbar\left[H'_{11} + H'_{22} - \hbar\omega_{12}\right]}{2\hbar^2} \qquad (34)$$

The Hamiltonian part of the perturbing internal field B_i which interact with spin $\underline{S}$ is given

$$H' = \frac{e}{2m} S_z Bi \qquad (35)$$

Where

$$Bi = \frac{\mu_\circ i}{2r} = \frac{\mu_\circ nev}{2r} = \frac{\mu_\circ nep}{2ma} \qquad (36)$$

$\mu_\circ$ = magnetic permeability in free space, i= current.

The corresponding proton is given by:

$$Bi = \frac{\mu_\circ ne}{2ma}\frac{\hbar\vec{\nabla}}{i} = \frac{\mu_\circ ne\widehat{p}}{2ma} \quad (37)$$

Where:

r=a=atomic radius, $\hat{p}$ momentum operator using Heisenberg picture:

$$i\hbar\hat{p} = mi\hbar\frac{d\hat{x}}{dt} = m\left[\hat{X},\hat{H}\right] \quad (38)$$

Hence

$$H' = \frac{e^2 S_z}{2ma}\frac{\mu_0 n\left[\hat{X},\hat{H}\right]}{i\hbar} \quad (39)$$

$$H'_{mn} = \int \bar{u}_m H' u_n dx = \frac{e^2\mu_\circ ns_z}{4ma\hbar i}\int \bar{u}_m\left[\hat{X}\hat{H} - \hat{H}\hat{X}\right]u_n dx$$

$$H'_{mn} = \frac{e^2\mu_0 ns_z}{4ma\hbar i}\left\{\int \bar{u}_m\hat{x}E_n u_n dx - \int \bar{u}_m\hat{H}\hat{\chi}u_n dx\right\}$$

$$= \frac{e^2\mu_0 ns_z}{4ma\hbar i}\left[E_n\int \overline{u_m}\hat{\chi}u_n dx - \int \hat{H}\overline{u_m}\hat{x}u_n dx\right]$$

$$= \frac{e^2\mu_\circ ns_z}{4ma\hbar i}\left[(E_n - E_m)\int \bar{u}_m\bar{x}u_n dx\right]$$

$$H'_{mn} = \frac{e^2\mu_\circ nS_z\hbar}{4ma\hbar i}\omega_{nm}\mu_{mn} = \frac{e^2\mu_\circ nS_z}{4mai}\omega_{nm}\mu_{mn} \quad (40)$$

Where

$$E_n - E_m = \hbar\omega_n - \hbar\omega_m = \hbar\omega_{nm}$$

$$\mu_{mn} = \int \bar{u}_m\hat{x}u_n dx \quad (41)$$

And the hermilicity of $\hat{H}$ requires

$$\int \bar{u}_\omega\hat{H}\hat{x}u_n dx = \int \overline{\hat{H}u_m}u_n dx = E_m\int \bar{u}_m\hat{x}u_n dx = E_m\mu_{mn} \quad (42)$$

But according to modified Schrödinger equation [18]

$$\omega = \frac{k}{\sqrt{\mu_\beta\varepsilon_\circ\mu_r\varepsilon_r}} - \frac{i\sigma}{2\varepsilon_\circ\varepsilon_r}\cdot\sigma = \chi_z\omega \quad (43)$$

Where:

K=wave number, $\varepsilon_\circ$ =electric permissibility in free space, ε_r = relative electric permissibility, μ_B = Boher magnetron

μ_r = relative magnetic permittivity

$$\omega = \frac{\omega_\circ}{\sqrt{\mu_r\varepsilon_r}} - \frac{i\chi_z\omega_\circ}{2\varepsilon_\circ\varepsilon_r} \quad (44)$$

$$\omega = c_r\omega_\circ - i\chi_r\omega_\circ$$

With

$$C_r = \frac{1}{\sqrt{\mu_r\varepsilon_r}} \qquad \chi_r = \frac{\chi_z}{2\varepsilon_\circ\varepsilon_r} \quad (45)$$

C_r = v/c

There for: v=medium speed of light, c= free space speed of light

Thus

$$E_n = C_r \hbar \omega_n - i x_r \hbar \omega_n$$

$$\hbar \omega_{nm} = c_r \hbar \omega_{nm} - i \chi_r \hbar \omega_{nm} \quad (46)$$

Thus according to equations (40) and one gets

$$H'_{22} = \frac{e^2 \mu_\circ n s_z \omega_{22} \mu_{22}}{4mai} = 0$$

$$\hbar \omega_{12} = c_r \hbar \omega_{12} - i \chi_r \hbar \omega_{12} \quad (47)$$

χ_r =relative magnetic susceptibility

Sub. (47) in (10) yields:

$$b = i c_r \hbar^2 \omega_{12} + \chi_r \hbar^2 \omega_{12} \quad (48)$$

In new of eq. (48)

Relation (26) reads:

$$\beta = i \frac{c_r \omega_{12}}{2} + \frac{\chi_r \omega_{12}}{2} \quad (49)$$

Utilizing equation (10) together with (18), (40) one gets

$$C = H'_{12} H'_{21} - H'_{11} H'_{22} + \hbar \omega_{12} H'_{11}$$

$$C = H'_{12} H'_{21}$$

$$= \left[\frac{e^2 \mu_\circ n s_z}{4mai\hbar} \right]^2 (\hbar \omega_{12} \mu_{12})(\hbar \omega_{21} \mu_{21})$$

$$= -\left[\frac{e^2 \mu_\circ n s_z}{4ma\hbar} \right]^2 (\mu_{12} \mu_{21}) \left[(c_r - i\chi_r)(\hbar \omega_{12})(c_r - i\chi_r)\hbar \omega_{21} \right]$$

$$= -\left[\frac{e^2 \mu_\circ n s_z}{4ma\hbar} \right]^2 (\mu_{12} \mu_{21})(\hbar^2 \omega_{12} \omega_{21}) \left[c_r^2 - c_r^2 \chi_r^2 - 2i \chi_r c_r \right]$$

(50)

One can simplify the above expression by bearing in mind that:

$$C = \frac{1}{\sqrt{\mu_\circ \varepsilon_\circ}} \; v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_\circ \varepsilon_\circ} \sqrt{\mu_r \varepsilon_r}}$$

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \; \frac{c}{v} = \sqrt{\mu_r \varepsilon_r} > 1 \quad (51)$$

$$\chi_z = \frac{n_1 \alpha}{2\pi c \mu \omega} \qquad \text{[rev.18]} \qquad (52)$$

$$n_1 \approx 1 \;, \quad \alpha \approx 10^{-5} \,, \; C \approx 10^8 \;, \; \mu \approx 10 \,, \quad \omega \approx 10^3 \quad (53)$$

$$\chi_z \approx \frac{10^{-5}}{10^8 \times 10^{=7} \times 10^3} \approx 10^{-9} \quad , \quad \varepsilon_\circ \approx 10^{-12} \,, \quad \varepsilon_r \approx 1$$

n_1 =Radio wave frequency refractive frequency, $\alpha = \frac{\omega}{c} n_2$, $n_2 = \frac{c^2 \mu \varepsilon_2}{n_1}$

$$\chi_r = \frac{\chi_z}{2\varepsilon_\circ \varepsilon_r} \quad (54)$$

$$\chi_r \approx \frac{10^{-9}}{2 \times 10^{-12} \times 1} \qquad \chi_r \approx 500$$

When $C_R > \chi_r$

(47),(48),(49),(50) and (27) can be simplified to get:

$$b = ic_r \hbar^2 \omega_{12},$$

$$C = -\left[\frac{e^2 \mu_\circ n s_z}{4ma\hbar}\right]^2 [\mu_{12}\mu_{21}][\hbar^2 \omega_{12}\omega_{21}] C_r^2 \quad (55)$$

$$\beta = \frac{ic_r \omega_{12}}{2}$$

$$\omega = c_r \sqrt{-\left[\frac{e^2 \mu_\circ n s_z}{4ma\hbar}\right]^2 \mu_{12}\mu_{21}\omega_{12}\omega_{21} + \frac{\omega_{12}^2}{4}}$$

$$\omega = \frac{C_r}{2}\sqrt{\omega_{12}^2 - \left[\frac{e^2 \mu_\circ n s_z}{ma\hbar}\right]^2 \mu_{12}\mu_{21}\omega_{12}\omega_{21}}$$

$$\omega = {}^{i}\sqrt{\frac{[e^2 \mu_\circ n s_z]^2 \mu_{12}\mu_{21}\omega_{12}\omega_{21} - \omega_{12}^2 = i\gamma_\circ}{ma\hbar}} \quad (56)$$

Yield:

$$C_1 = Ae^{-\gamma_0 t} e^{-i\frac{c_r \omega_{12r}}{2}t} \quad (57)$$

Taking the real part of C yields:

$$C_1 = Ae^{-\gamma_{9\circ} t} \cos\frac{(c_r \omega_{12} t)}{2}$$

$$C_1^2 = A^2 e^{-2\gamma_{0\circ} t} \cos^2 \frac{(c_r \omega_{12} t)}{2} \quad (58)$$

Thus the transverse relaxation time is given from the expositional term as:

$$T_2 = \frac{1}{2\gamma_\circ =} = -\frac{ma\hbar}{2\sqrt{(e^2 \mu_\circ n s_z)^2 \mu_{12}\mu_{21}\omega_{12}\omega_{21} - m^2 a^2 \hbar^2 \omega_{12}^2}} \quad (59a)$$

$$\frac{c_r \omega_{12} T_2}{2} = \frac{\pi}{2} \qquad T_2 = \frac{\pi}{c_r \omega_{12}} = \frac{\pi}{c_r \omega}$$

This mean:

$$T_2 \; \alpha \; \frac{1}{\omega} \qquad (59b)$$

Where:

$$\hbar\omega = E_1 - E_2 = \hbar(\omega_1 - \omega_2) = \hbar\omega_{12}$$

If interaction potential is considered as a number where:

$$H' = \frac{e}{2m} S_Z B_i \qquad (60)$$

S_z = spin momentum in z- axis

$$H'_{mn} = \int \bar{u}_m H' u_n dr = \frac{e}{2m} S_Z B_i \int \bar{u}_m u_n dr$$

$$H'_{mn} = \frac{e}{2m} S_Z B_i \delta_{nm} \qquad (61)$$

Two level system n, m=1.2

$$H'_{11} = H'_{22} = \frac{e}{2m} S_Z B_i$$
$$H'_{12} = H'_{21} = 0 \qquad (62)$$

Thus according to equation (10)

$$a = -\hbar^2 \qquad b = i\hbar\left[\frac{e}{m} S_z B_I - \hbar\omega_{12}\right]$$

$$C = \frac{-e^2 s_z B_i^2}{4m^2} + \frac{e\hbar S_z}{2m} B_i \omega_{12} \qquad (63)$$

Thus according to equations (26) and (27), are gets

$$\beta = \frac{i\hbar e S_Z B_i}{2m\hbar} - \frac{i\hbar^2 \omega_{12}}{2\hbar^2}$$
$$\beta = \frac{i}{2\hbar}\left[\frac{e}{m} S_z B_i - \hbar\omega_{12}\right] \qquad (64)$$

Hence

$$\left[\omega = \frac{1}{2}\sqrt{\frac{-e^2 S_z^2 B_i^2}{m^2} + \frac{2e\hbar S_z B_i \omega_{12}}{m} + \left(\frac{e}{m} S_z B_i - \hbar\omega_{12}\right)^2}\right] \qquad (65)$$

For very small mass and very strong internal magnetic field equation (64) and becomes:

$$\omega \approx \sqrt{-\frac{e^2 s_z^z \beta_i^z}{m^2}} \qquad = \mathrm{i}\,\frac{e}{m} S_z B_i \qquad (66)$$

$$\beta = \frac{i}{2\hbar}\frac{e}{m} S_z B_i \qquad (67)$$

Substituting (66) and (67) in (21) yields:

$$C_1 = A\, e^{\frac{e s_z B_i}{m}} \; e^{\frac{-i e s_z B_i}{2\hbar m}}$$

$$C_1 = \frac{Ae^{\frac{es_z B_i}{m_i}}}{e^{\frac{iesB}{2\hbar m}}}$$

Taking the real part

$$C_1 = \frac{Ae^{\frac{es_z B_i}{m_i}}}{\cos es_z B_i t / 2m\hbar} \qquad (68)$$

$$|C_1|^2 = \frac{e^{\frac{2\ es_z B_i}{m_i}}}{\cos^2 \frac{es_z B_i t}{2m\hbar}} \qquad (69)$$

With the aid of equation (4.4.69) the exponential term gives

$$T_2 = t = \frac{m}{2es_z B_i} \qquad (70)$$

And the cosine term gives

$|C_1|^2 = 1$ At t=0 $|C_1|^2 = 0$

$$\frac{es_z B_i T_2}{2m\hbar} = \frac{\pi}{2}$$

$$T_2 = \frac{m\hbar\pi}{es_z B_i} \qquad (71)$$

When the internal field B_i is neglected equations (64) and (65) reads

$$\beta = \frac{-i\omega_{12}}{2}$$

$$\omega = \frac{1}{2}\sqrt{(\hbar\omega_{12})^2} = \frac{\hbar\omega_{12}}{2}$$

In view of equation (21)

$$C_1 = Ae^{\frac{i\omega_{12}t}{2}} e^{\frac{-\ell\omega_{12}t}{2}}$$

Taking the real part of C_1 thus:

$$C_1 = Ae^{-\hbar\frac{\omega_{12}t}{2}} \cos\omega_{12} \, t/2$$

$$|C_1|^2 = A^2 e^{-\hbar\omega_{12}t} \cos^2 \frac{\omega_{12}t}{2}$$

Thus the relaxation time according to the exponential term is given by:

$$T_2 = t = \frac{2}{\hbar\omega_{12}}$$

And according to the cosine term is given by

$|C_1|^2 = 1$ t=0

$|C_1|^2 = 0$ $\frac{\omega_{12}}{2} t = \frac{\pi}{2}$

$$T_2 = \frac{\pi}{\omega_{12}} = \frac{\pi}{\omega_1 - \omega_2} = \frac{\pi}{\omega} \quad (72)$$

Where is ω the proton frequency which is readied for transition from level

$E_2 = \hbar\omega_2$ to level $E_1 = \hbar\omega_1$ such that [19,20, 21, 22, 23]

$$\hbar\omega = E_1 - E_2 = \hbar\omega_1 - \hbar\omega_2$$

II. DISCUSSION

Schrodinger Equation in the energy space two level systems in equation (1) is used to find new expressions for the relaxation time. The energy wave function was assumed to be in the form suggested in equation (12). Substituting this solution in the Schrodinger equation (11) and equating the coefficients of sin and cos the frequency ω was found in equation (18).Using the fact that the square of the wave function C_1 is proportional to probability the relaxation time is shown to be inversely proportional to angular frequency as equation (20) shows. This agrees with the ordinary one. For electric interaction another exponential solution of C_1 was suggested in equation (21). Using this in equation (11) and equating real and imaginary parts, one founds ω in (27). Bearing in mind that $|C_1|^2$ is the probability in equation (31) and (32) the relaxation time T_2, i.e. the transverse one, is dependent on electric moment as equation (59a) indicates. When the interaction potential depends on the magnetic interaction, the transverse relaxation time depends on the internal magnetic field, which conforms to the ordinary one, since it depends generally on the spin.

III. CONCLUSION

Using Schrodinger equation in the energy space a useful expression for the transverse relaxation time was found. It was shown that, when electric interaction dominates the transverse relaxation time depends on electric dipole moment. But when magnetic interaction becomes important it depends on internal magnetic field as well as magnetic spin.

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